

# Independence and Bayesian Networks (Part 2)

Wenhu Chen

Lecture 4

# Outline

Learning Goals

D-Separation

Constructing Bayesian Networks

Causality

Revisiting Learning Goals

# Learning Goals

- ▶ Determine whether an independence relationship holds by applying d-separation.
- ▶ Given a Bayesian network and an order of the variables, construct a Bayesian network that correctly represents the independence relationships among the variables.
- ▶ Understand the difference between correlation and causality.

Learning Goals

**D-Separation**

Constructing Bayesian Networks

Causality

Revisiting Learning Goals

# D-Separation

Are two variables  $X$  and  $Y$  independent given the set of observed variables  $E$ ?

## Definition (D-Separation)

$E$  d-separates  $X$  and  $Y$

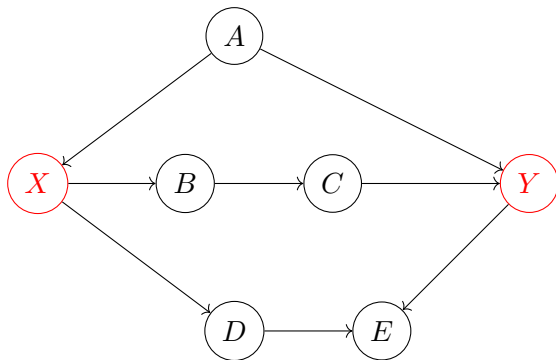
iff  $E$  blocks every un-directed path between  $X$  and  $Y$ .

If  $E$  d-separates  $X$  and  $Y$ ,

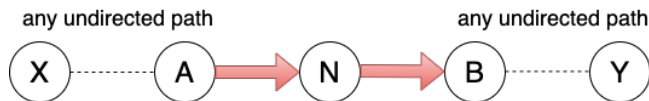
then  $X$  and  $Y$  are conditionally independent given  $E$ .

## D-Separation

- ▶ Un-directed paths between  $X$  and  $Y$ .
- ▶ Multiple paths need to be considered if they exist.
- ▶ One of the nodes on all the paths blocking the connection.



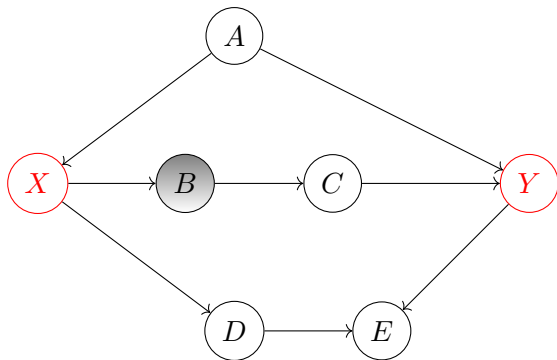
## Blocked Path - Scenario 1/3



If N is observed,  
then it blocks the path between X and Y.

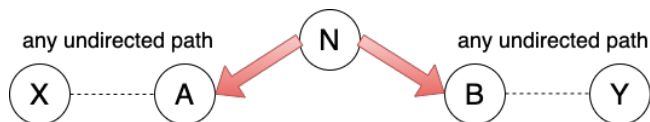
## D-Separation (Scenario 1/3)

- ▶  $B$  blocks the path between  $X$  and  $Y$ , which follows scenario 1.





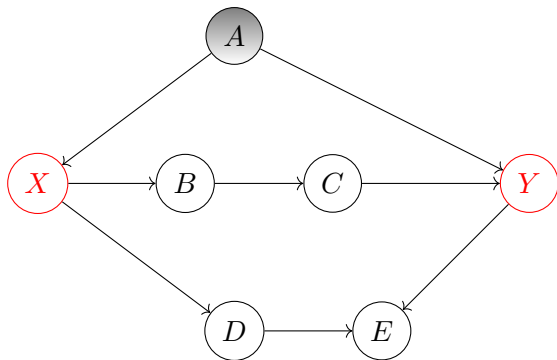
## Blocked Path - Scenario 2/3



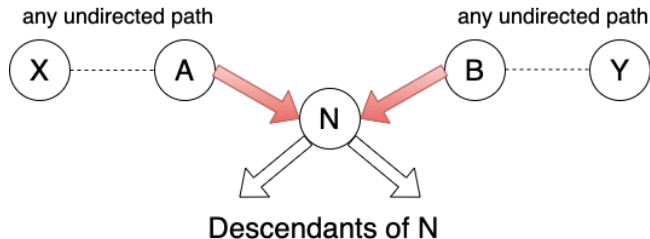
If N is observed,  
then it blocks the path between X and Y.

## D-Separation (Rule 2)

- ▶  $A$  blocks the path between  $X$  and  $Y$ , which follows scenario 2.



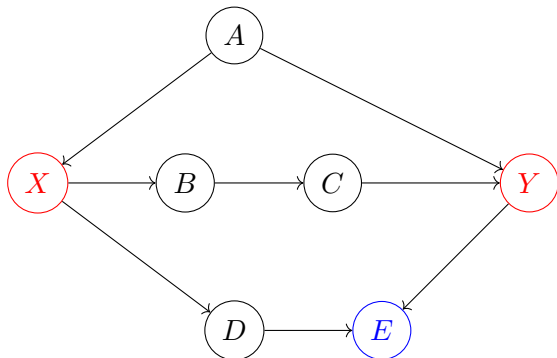
## Blocked Path - Scenario 3/3



If N and N's descendants are NOT observed,  
then they block the path between X and Y.

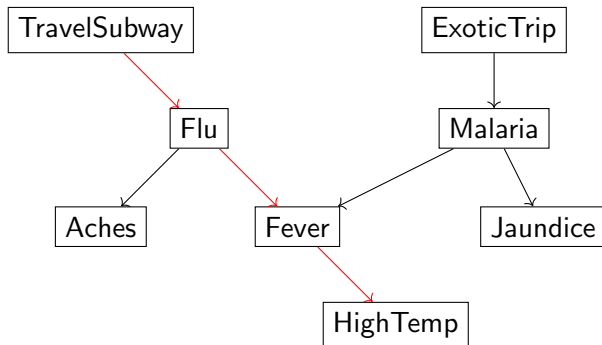
## D-Separation (Rule 3)

- ▶  $E$  blocks the path between  $X$  and  $Y$ , which follows scenario 3.
- ▶ If  $E$  is not observed, then  $X$  and  $Y$  are d-separated.



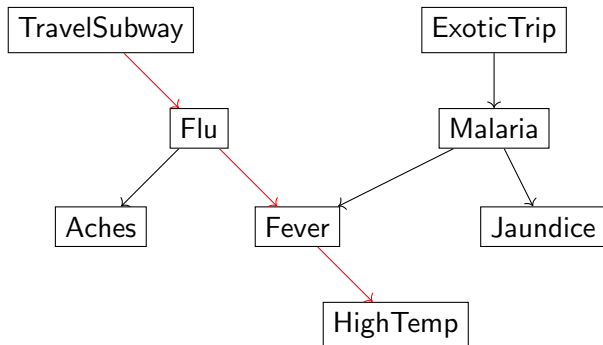
## Q: Applying D-separation

Q #1: Are **TravelSubway** and **HighTemp** independent?



## Q: Applying D-separation

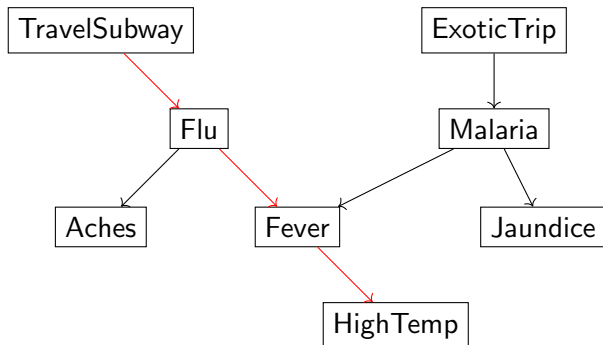
Q #1: Are **TravelSubway** and **HighTemp** independent?



- ▶ A path from TravelSubway to Flu to Fever to HighTemp.
- ▶ Two nodes on the path: Flu and Fever.
- ▶ Apply rule 1 to Flu, Flu is not observed, no blocking
- ▶ Apply rule 1 to Fever, Fever is not observed, no blocking

## Q: Applying D-separation

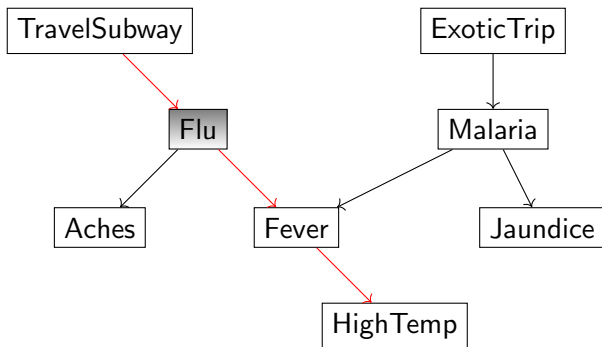
**Q #1:** Are **TravelSubway** and **HighTemp** independent?



→ No, they not independent.

## Q: Applying D-separation

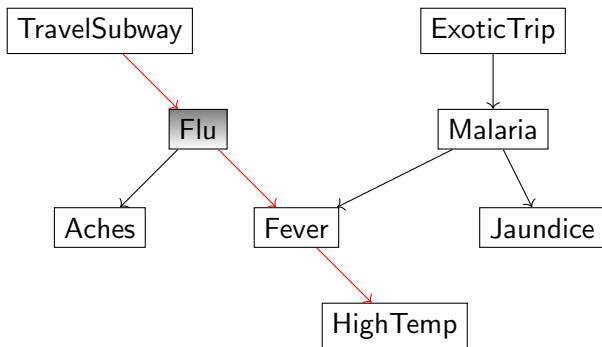
Q #2: Are **TravelSubway** and **HighTemp** independent given **Flu**?





## Q: Applying D-separation

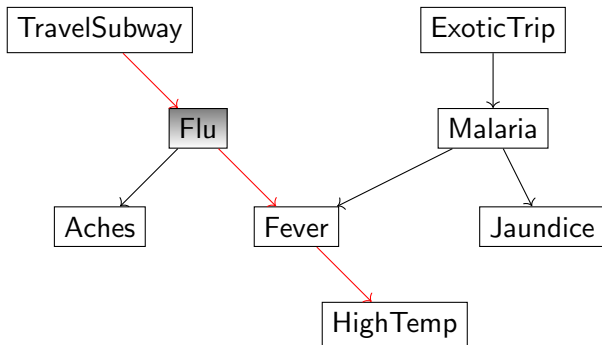
Q #2: Are **TravelSubway** and **HighTemp** independent given **Flu**?



- ▶ A path from TravelSubway to Flu to Fever to HighTemp.
- ▶ Flu and Fever are the nodes on the path.
- ▶ Apply rule 1 to Flu, Flu is observed, the path is blocked.

## Q: Applying D-separation

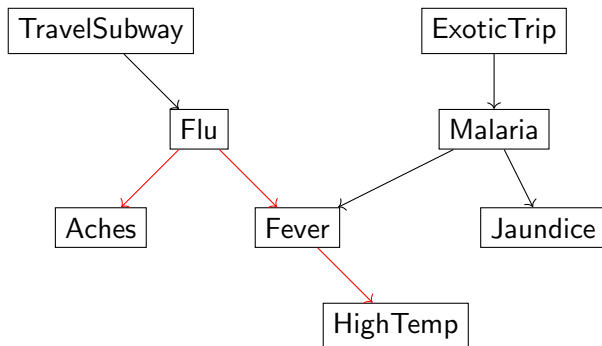
Q #2: Are **TravelSubway** and **HighTemp** independent given **Flu**?



→ Yes, they are independent given Fever

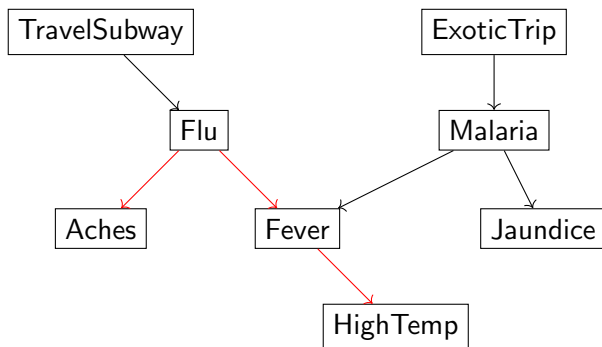
## Q: Applying D-separation

**Q #3:** Are **Aches** and **HighTemp** independent?



## Q: Applying D-separation

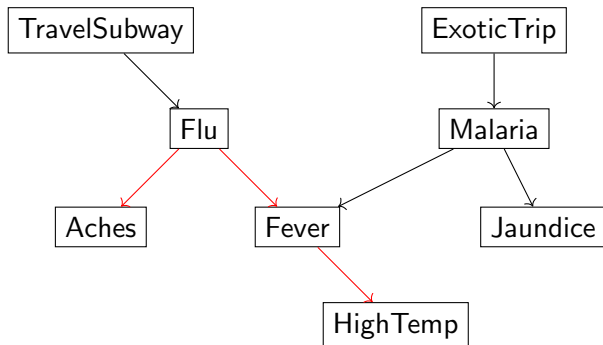
**Q #3:** Are **Aches** and **HighTemp** independent?



- ▶ One path from Aches to Flu to Fever to High Temp
- ▶ Apply Rule 1 to Fever, Fever is not observed, no blocking
- ▶ Apply Rule 2 to Flu, Flu is not observed, no blocking

## Q: Applying D-separation

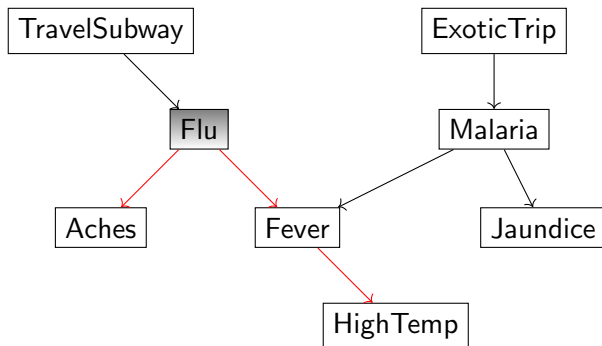
**Q #3:** Are **Aches** and **HighTemp** independent?



→ No, they are not independent.

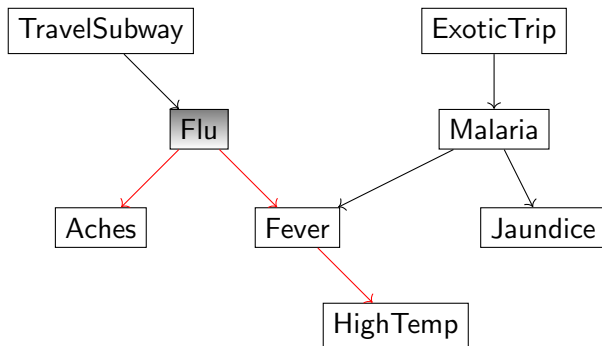
## Q: Applying D-separation

**Q #4:** Are **Aches** and **HighTemp** independent given **Flu**?



## Q: Applying D-separation

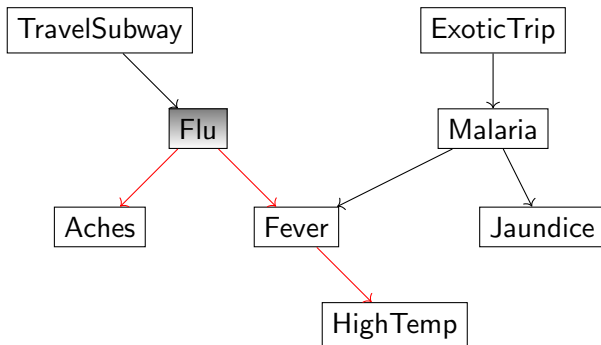
**Q #4:** Are **Aches** and **HighTemp** independent given **Flu**?



- ▶ One path from Aches to Flu to Fever to High Temp.
- ▶ Apply Rule 1 to Fever, Fever is not observed, no blocking.
- ▶ Apply Rule 2 to Flu, Flu is observed, and the path is blocked.

## Q: Applying D-separation

**Q #4:** Are **Aches** and **HighTemp** independent given **Flu**?

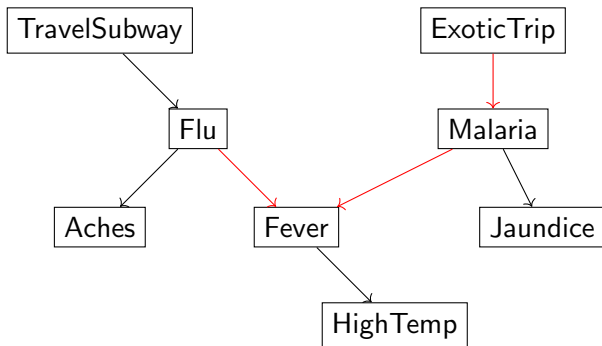


→ Yes, they are independent given Flu.



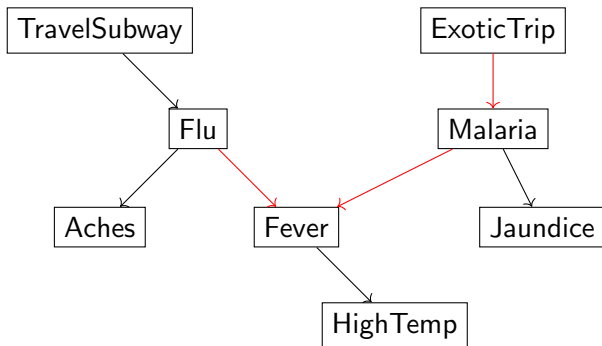
## Q: Applying D-separation

Q #5: Are **Flu** and **ExoticTrip** independent?



## Q: Applying D-separation

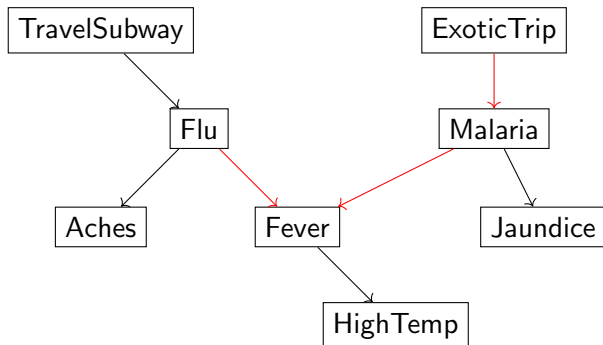
Q #5: Are **Flu** and **ExoticTrip** independent?



- ▶ One path from Flu to Fever to Malaria to ExoticTrip.
- ▶ Apply rule 1 to Malaria, which is not observed, not blocking.
- ▶ Apply rule 3 to Fever, itself and descendent are not observed, the path is being blocked.

## Q: Applying D-separation

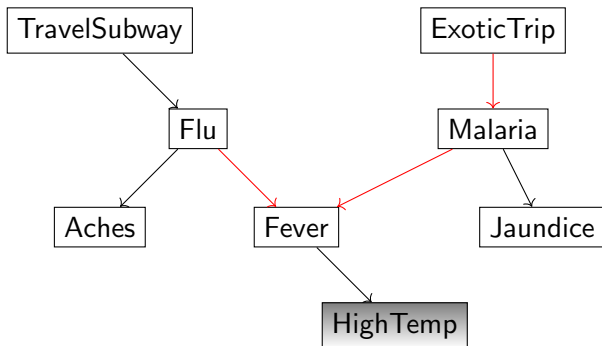
**Q #5:** Are **Flu** and **ExoticTrip** independent?



→ Yes, they are indepent.

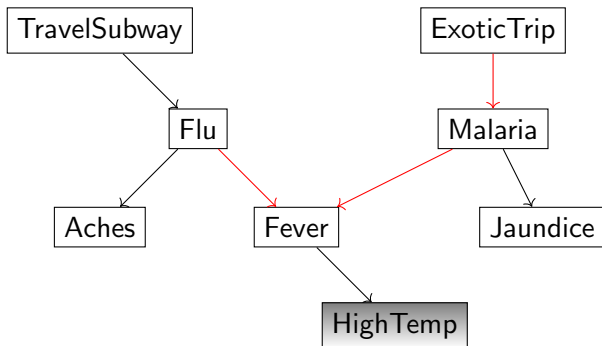
## Q: Applying D-separation

**Q #6:** Are **Flu** and **ExoticTrip** independent given **HighTemp**?



## Q: Applying D-separation

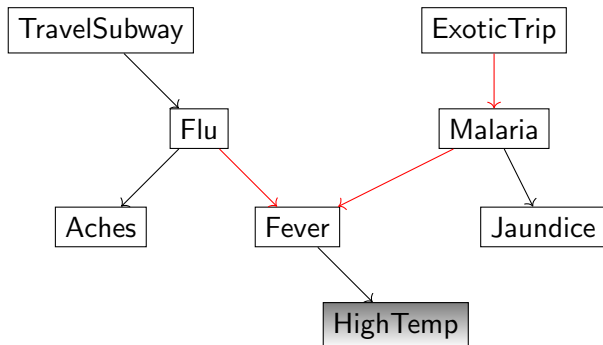
**Q #6:** Are **Flu** and **ExoticTrip** independent given **HighTemp**?



- ▶ One path from Aches to Flu to Fever to High Temp.
- ▶ Apply rule 1 to Fever, Fever is not observed, no blocking.
- ▶ Apply rule 3 to Fever, its descendent is observed, the path is not being blocked.

## Q: Applying D-separation

**Q #6:** Are **Flu** and **ExoticTrip** independent given **HighTemp**?



→ They are not independent given HighTemp.

Learning Goals

D-Separation

Constructing Bayesian Networks

Causality

Revisiting Learning Goals

# Constructing Bayesian Networks

- ▶ For a joint probability distribution, there are many correct Bayesian networks.
- ▶ Given a Bayesian network  $A$ , a Bayesian network  $B$  is correct if and only if the following is true:

*If Bayesian network  $B$  requires two variables to satisfy an independence relationship, Bayesian network  $A$  must also require the two variables to satisfy the same independence relationship.*

- ▶ Bayesian network  $B$  could miss independence from Network  $A$ , but it cannot miss dependence.
- ▶ We prefer a Bayesian network that requires fewer probabilities.



# Constructing a Correct Bayesian Network

1. Order the variables  $\{X_1, \dots, X_n\}$ .
2. For each variable  $X_i$  in the ordering,

## 2.1 Choose the node's parents:

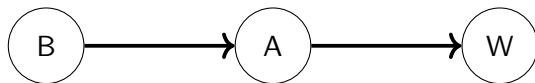
Choose the smallest set of parents from  $\{X_1, \dots, X_{i-1}\}$  such that given  $Parents(X_i)$ ,  $X_i$  is independent of all the nodes in  $\{X_1, \dots, X_{i-1}\} - Parents(X_i)$ . Formally,

$$P(X_i | Parents(X_i)) = P(X_i | X_{i-1} \wedge \dots \wedge X_1).$$

- 2.2 Create a link from each parent of  $X_i$  to the node  $X_i$ .
- 2.3 Write down the conditional probability table  $P(X_i | Parents(X_i))$ .

## Example 1: Construct a Bayes Net

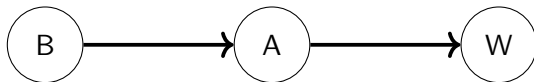
Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: W, A, and B.

## Example 1: Construct a Bayes Net

Consider the Bayesian network.



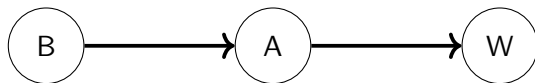
Construct a correct Bayesian network by adding the variables in the order: W, A, and B.

Set: {}



## Example 1: Construct a Bayes Net

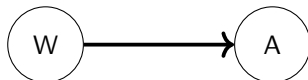
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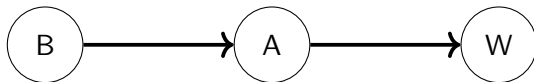
Set: {W}

- ▶ Is A dependent on W?



## Example 1: Construct a Bayes Net

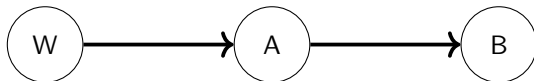
Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: W, A, and B.

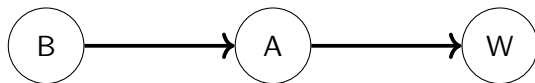
Set:  $\{W, A\}$

- ▶ Is B independent from A given W?
- ▶ Is B independent from W given A?



## Example 1: Construct a Bayes Net

Consider the Bayesian network.



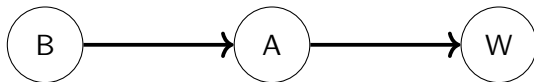
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## Example 1: Construct a Bayes Net

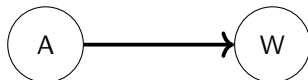
Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: A, W, and B.

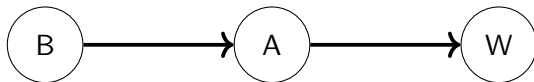
Set: {A}

- ▶ is W dependent on A?



## Example 1: Construct a Bayes Net

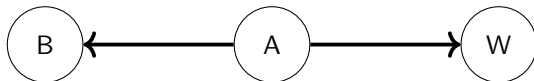
Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: A, W, and B.

Set:  $\{A, W\}$

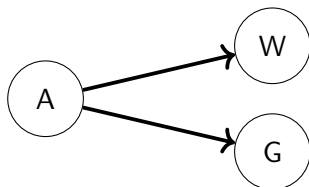
- ▶ B is independent from A given W?
- ▶ B is independent from W given A?





## Example 2: Construct a Bayes Net

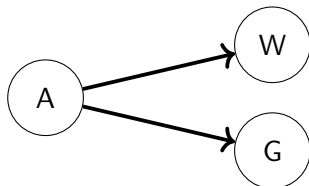
Consider the Bayesian network:



Construct a correct Bayesian network by adding the variables in the order: W, G, and A.

## Example 2: Construct a Bayes Net

Consider the Bayesian network:

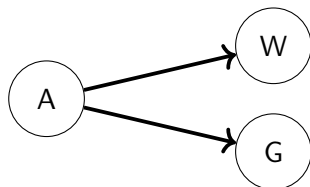


Set: {}



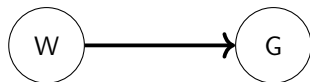
## Example 2: Construct a Bayes Net

Consider the Bayesian network:



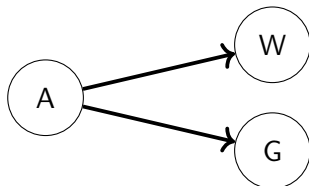
Set:  $\{W\}$

- ▶ Is G dependent on W?



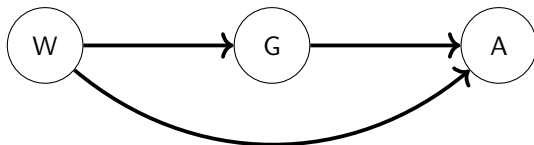
## Example 2: Construct a Bayes Net

Consider the Bayesian network:



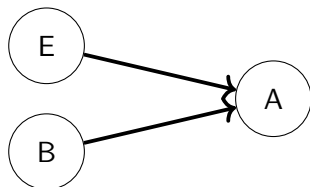
- ▶ Is A independent on G given W?
- ▶ Is A independent on W given G?

Set: {W, G}



## Example 3: Construct a Bayes Net

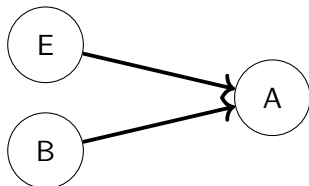
Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: A, B, and E.

## Example 3: Construct a Bayes Net

Consider the Bayesian network.

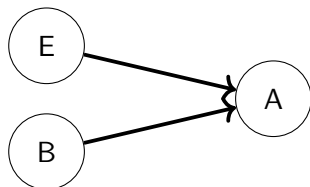


set:  $\{\}$



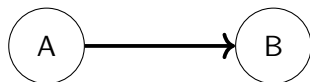
## Example 3: Construct a Bayes Net

Consider the Bayesian network.



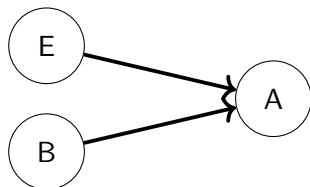
set: {A}

- ▶ Is B dependent on A?



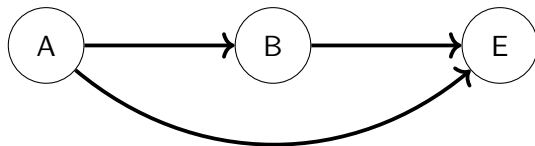
## Example 3: Construct a Bayes Net

Consider the Bayesian network.



set:  $\{A, B\}$

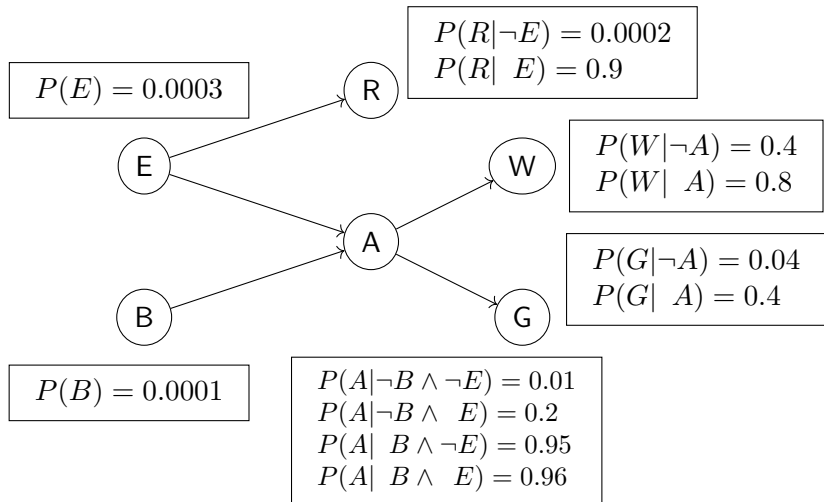
- ▶ Is E independent on A given B?
- ▶ Is E independent on B given A?





## Example 3: Reconstruct the Holmes Scenario Network

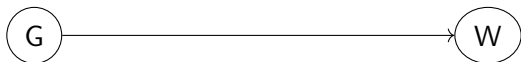
Construct a new Bayesian network from the Holmes scenario, using the following order for adding variables:  $G, W, E, B, A, R$ .



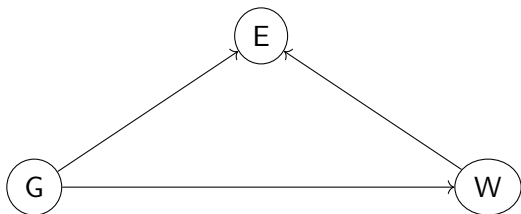
## Example 3: Reconstruct the Holmes Scenario Network



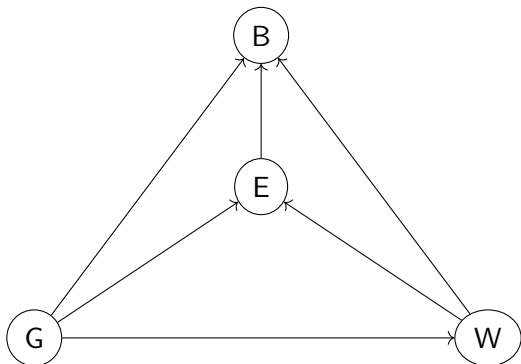
## Example 3: Reconstruct the Holmes Scenario Network



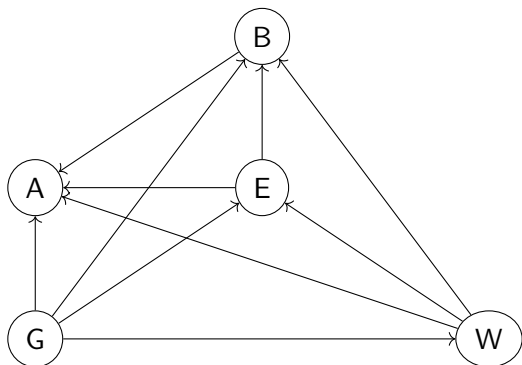
## Example 3: Reconstruct the Holmes Scenario Network



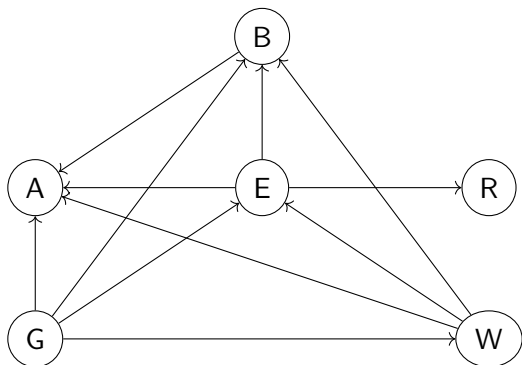
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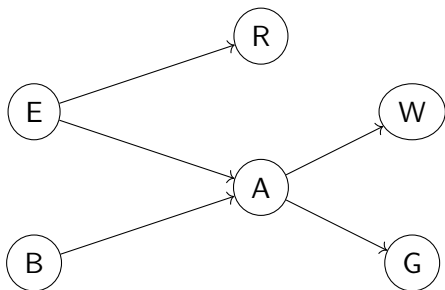


## Example 3: Reconstruct the Holmes Scenario Network

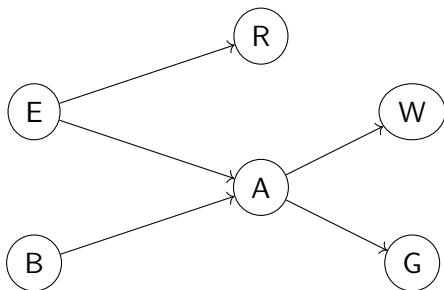
- ▶ Gibbon and Watson can both cause Earthquake.
- ▶ Gibbon and Watson and Earthquake can cause Burglary.
- ▶ Number of probabilities:  $1 + 2 + 4 + 8 + 16 + 2 = 33$ .
- ▶ Previous, we only need 12 probabilities.



## What is the correct order of correct Reconstruction



## What is the correct order of correct Reconstruction



- ▶ Correct Order: E, R, B, A, W, G, Let's validate it!
- ▶ Finding the most compact Bayesian Network is NP-hard!

# Constructing a Compact Bayesian Network

- ▶ What does an edge mean?

Does an edge always represent a causal relationship?

→ An edge indicates an associational relationship that is not necessarily causal.

- ▶ How can we construct a Bayesian network with the smallest number of edges?

→ Cause precedes effect. So add causes first, then effects.

Learning Goals

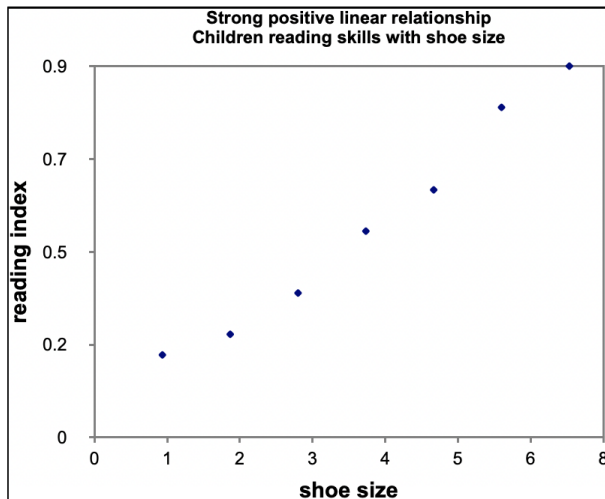
D-Separation

Constructing Bayesian Networks

Causality

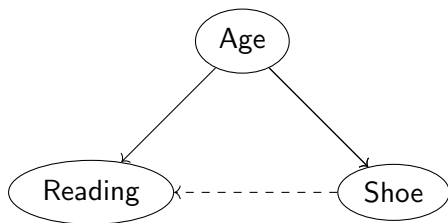
Revisiting Learning Goals

## Are Correlation and Causation the same?



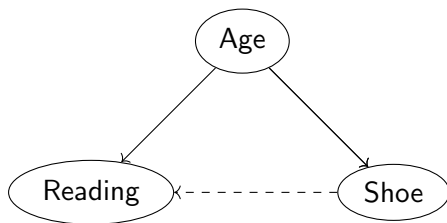
Why are these two factors so highly correlated?

## Causality vs Correlation



- ▶ There is a confounding variable 'Age', which we did not take into account.
- ▶ The hidden variable confounds the relationship between Shoe Size and Reading.
- ▶ Randomized Experiments (Causal Intervention).

# Causal Intervention



- ▶ Intervention:  $p(R|do(Shoe) = 1) - p(R|do(Shoe) = 0)$ .
- ▶ Average Treatment Effect:  
 $ATE = \sum_A p(R|S = 1, A)p(A) - \sum_A p(R|S = 0, A)p(A)$
- ▶  $ATE \approx 0$ , which means no causal relation.

Reference: <https://www.bradyneal.com/slides/1%20-%20A%20Brief%20Introduction%20to%20Causal%20Inference.pdf>

# Revisiting Learning Goals

- ▶ Determine whether an independence relationship holds by applying d-separation.
- ▶ Given a Bayesian network and an order of the variables, construct a Bayesian network that correctly represents the independence relationships among the variables.
- ▶ Understand the difference between correlation and causality.