Lecture 10: Reinforcement Learning
CS486/686 Intro to Artificial Intelligence

2024-6-11

Pascal Poupart
David R. Cheriton School of Computer Science
CIFAR AI Chair at Vector Institute

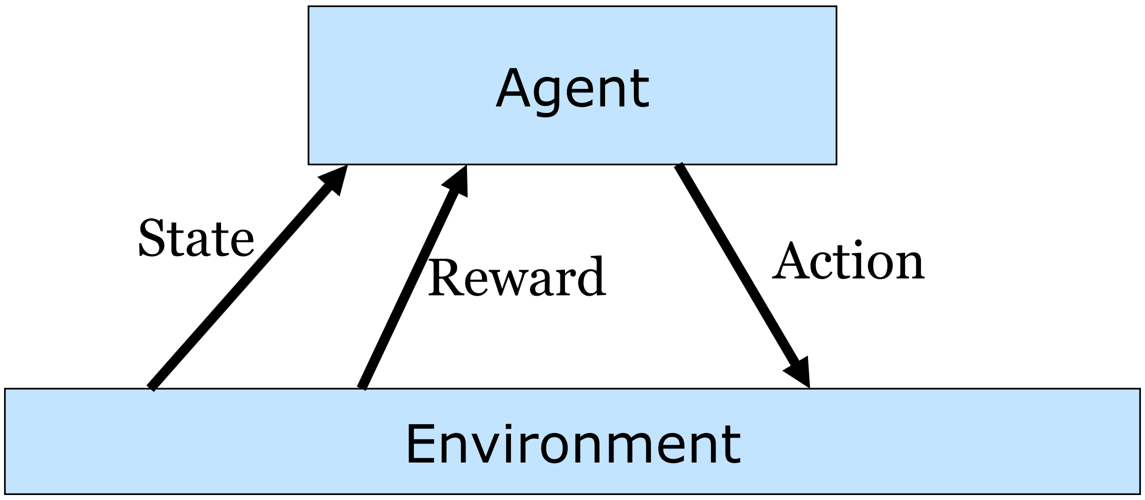
UNIVERSITY OF WATERLOO

Outline

- Reinforcement Learning
  - Q-Learning
  - Exploration strategies

UNIVERSITY OF WATERLOO

Recap: Reinforcement Learning Problem



Goal: Learn to choose actions that maximize rewards

Reinforcement Learning

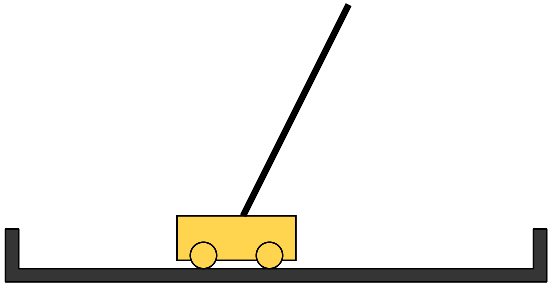
- Formal Definition
  - States: $s \in S$
  - Actions: $a \in A$
  - Rewards: $r \in \mathbb{R}$
  - ~~Transition model:~~ $\cancel{\Pr(s_t|s_{t-1}, a_{t-1})}$
  - ~~Reward model:~~ $\cancel{\Pr(r_t|s_t, a_t)}$
  - Discount factor: $0 \leq \gamma \leq 1$
  - Horizon (i.e., # of time steps): h

unknown

- Goal: find optimal policy $\pi^* = argmax_\pi \sum_{t=0}^{h} \gamma^t E_\pi[r_t]$

Example: Inverted Pendulum

- State: $x(t), x'(t), \theta(t), \theta'(t)$
- Action: Force F
- Reward: 1 for any step where pole balanced



Problem: Find $\pi: S \rightarrow A$ that maximizes rewards

Important Components in RL

RL agents may or may not estimate the following components:

- Model: $\Pr(s'|s,a)$, $\Pr(r|s,a)$
  - Environment dynamics and rewards
- Policy: $\pi(s)$
  - Agent action choices
- Value function: $V(s)$
  - Expected total rewards of the agent policy

UNIVERSITY OF WATERLOO

Categorizing RL agents

Value based
- No policy (implicit)
- Value function

Policy based
- Policy
- No value function

Actor critic
- Policy
- Value function

Model based
- Transition and reward model

Model free
- No transition and no reward model (implicit)

Online RL
- Learn by interacting with environment

Offline RL
- No environment
- Learn only from saved data

Bellman's Equation

- Value Iteration:

$$V_n^*(s) \leftarrow \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V_{n-1}^*(s')$$

- Bellman Equation (when $n \to \infty$):

$$V^*(s) = \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V^*(s')$$

- State-action Bellman Equation:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) \max_{a'} Q^*(s',a')$$

where $V^*(s) = max_a Q^*(s,a)$, $\pi^*(s) = argmax_a Q^*(s,a)$

Temporal Difference Control

- Approximate Q-function:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} \Pr(s'|s,a) \max_{a'} Q^*(s',a')$$
$$\approx r + \gamma \max_{a'} Q^*(s',a')$$

one sample approximation

- **Incremental update**

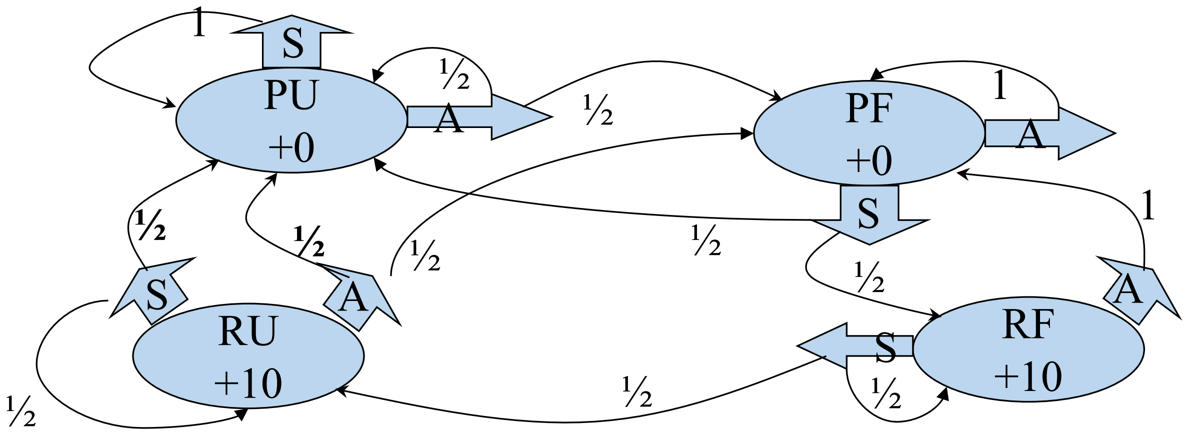
$$Q_n^*(s,a) \leftarrow Q_{n-1}^*(s,a) + \alpha_n \left(r + \gamma \max_{a'} Q_{n-1}^*(s',a') - Q_{n-1}^*(s,a) \right)$$

learning rate

Tabular Q-Learning

Qlearning()
  Initialize s and Q^* arbitrarily
  Repeat
    Select and execute a
    Observe s' and r
    Update counts: $n(s,a) \leftarrow n(s,a) + 1$
    Learning rate: $\alpha \leftarrow 1/n(s,a)$
    $Q^*(s,a) \leftarrow Q^*(s,a) + \alpha\left(r + \gamma \max_{a'} Q^*(s',a') - Q^*(s,a)\right)$
    $s \leftarrow s'$
  Until convergence of Q^*
Return Q^*

Q-learning Exercise



Current estimates:

$Q(RF,S) = 25$

$Q(RF,A) = 20$

$Q(RU,S) = 20$

$Q(RU,A) = 15$

Discount: $\gamma = 0.9$ Learning rate: $\alpha = 0.5$

Update $Q(RF,S)$ after executing S in RF and transitioning to RU:

$$Q(RF,S) \leftarrow Q(RF,S) + \alpha(r + \gamma \max\{Q(RU,S), Q(RU,A)\} - Q(RF,S))$$

$$\leftarrow 25 + 0.5(10 + 0.9 \max\{20, 15\} - 25)$$

$$\leftarrow 26.5$$

UNIVERSITY OF WATERLOO

Convergence

- Q-learning converges to optimal Q-values if
  - Every state is visited infinitely often (due to exploration)
  - The action selection becomes greedy as time approaches infinity
  - The learning rate α is decreased fast enough, but not too fast (sufficient conditions for α):

    (1) $\sum_t \alpha_t \to \infty$ (2) $\sum_t (\alpha_t)^2 < \infty$

- NB: $\alpha_t(s,a) = 1/n_t(s,a)$ satisfies the above conditions

Common Exploration Methods

- ϵ-greedy:
  - With probability ϵ, execute random action
  - Otherwise execute best action $a^* = argmax_a\, Q(s,a)$

- Boltzmann exploration
  - Increasing temperature T increases stochasticity

$$\Pr(a) = \frac{e^{\frac{Q(s,a)}{T}}}{\sum_a e^{\frac{Q(s,a)}{T}}}$$

UNIVERSITY OF WATERLOO